# Exam Statistical Methods in Physics
# Monday, April 11 2011, 9:00-12:00

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*Before you start, read the following:*

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- *Good luck!*

## Problem 1 (10 points)

Give a short and complete explanation to the following terms: (Avoid ambiguity in your understanding and use of similar terms)

(a) Define the term statistic. Is a statistic a random variable?

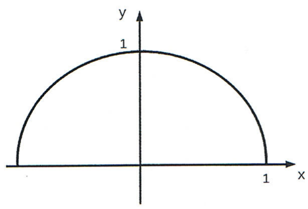
(b) Define the term "estimator." Is an estimator a random variable? What's the difference between an estimator and a statistic?

(c) Define the "standard error" of an estimator, $Q(x)$.

(d) What is the main motivation in choosing the critical region in hypothesis testing?

## Problem 2 (15 points)

A particle on the surface of a half-circular disk of unit radius as shown below.




The particle has a uniform probability density for being located at any position on the surface.

(a) Find $p(x, y)$, the joint probability density in the variables $x$ and $y$.

(b) Calculate $p(x)$, the probability density of the single random variable $x$. Make a plot of $p(x)$ with carefully labeled axes.

(c) Calculate $p(y)$ and make a plot of $p(y)$ with carefully labeled axes.

(d) Calculate $p(x|y)$, the conditional probability density of the random variable $x$ for a given value of $y$. Make a plot of $p(x|y)$ with carefully labeled axes.

(e) Let $r = x^2 + y^2$. Calculate $p(r)$ and make a plot of $p(r)$ with carefully labeled axes.

## Problem 3 (20 points)

Let $x_1, x_2, \ldots, x_n$ be a dataset that is a realization of a random sample from a distribution with probability density of $f_\delta(x)$ given by.

$$f_\delta(x) = \begin{cases} e^{-(x-\delta)} & \text{for } x \geq \delta \\ 0 & \text{for } x \leq \delta \end{cases}$$

(a) Determine $E[X]$ and $Var(X)$.

(b) Draw the likelihood $L(\delta)$.

(c) Determine the maximum likelihood estimate for $\delta$.

## Problem 4 (15 points)

You have a collection of 100 six-sided dice. For a single dice, the integers 1 through 6 are all equally likely to result after a roll. Suppose you roll all 100 dice at the same time. What is the approximate probability density for the sum $s$ (the summation of the values on all of the dice)?

Hint: Even though $s$ is discrete, write an expression for the continuous envelope function.

## Problem 5 (20 points)

Consider the following data set:

| x | y |
|---|---|
| 0.00 | 1.31 |
| 1.00 | 1.28 |
| 2.00 | 4.74 |
| 3.00 | 9.28 |
| 4.00 | 13.06 |

(a) Determine the least squares of estimate $\alpha$ of the parameter of the model $y = \alpha x$.

(b) Determine the least squares of estimate $\beta$ of the parameter of the model $y = \beta x^2$.

(c) Using the goodness of fit, explain which one of the two models is describing the data better (assume the error in measuring the variables is $\sigma = 1$)?

## Problem 6 (20 points)

You and your friends are considering to submit the solutions of this exam in some minutes(hopefully). There are many other students from other courses giving their exams right now and considering/going to submit their solutions in some minutes, too. In the last 5 minutes of exams, the number of submitted solutions in 1-minute interval follows a Poisson distribution with a rate of $\lambda$. Assume that the total number $X_i$ of submitted solutions during each of those 1-minute time intervals, will be sampled as the following data sample:

$$X = \{4, 2, 3, 1, 5\}$$

(a) Calculate the method of moments estimator for the rate $\lambda$ using the data sample.

(b) What is the probability that everybody submits their solutions before the last 5 minutes of the exams?

Now, instead of the number of submitted solutions, your exam's surveillance keeps track of the time spent waiting until you submit your solutions. His statistic textbook tells him that the waiting time $T_i$ for the $r^{th}$ occurrence of a Poisson event has a PDF of the following form:

$$f_T(t) = \begin{cases} \frac{\lambda^r}{(r-1)!} t^{(r-1)} e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

He knows that you submit your solutions always 3 positions earlier than the other students from other courses (i.e. $r = 3$) and he only keeps track of the time $T_i$ all the students are spending together until one of you submits his/her solution.

(c) What is the likelihood function for the sample $\{T_1, T_2, ..., T_n\}$ ?

(d) Find the maximum likelihood estimator for $\lambda$ (no number is required).

Table 1: Chi-squared distribution: $P(T \geq T_{obs}) = \int_{T_{obs}}^{\infty} \chi^2(NDF)dT$

| p value; NDF | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| .99 | 0.00 | 0.02 | 0.11 | 0.30 | 0.55 | 0.87 | 1.24 | 1.65 | 2.09 | 2.56 |
| .90 | 0.02 | 0.21 | 0.58 | 1.06 | 1.61 | 2.20 | 2.83 | 3.49 | 4.17 | 4.87 |
| .80 | 0.06 | 0.45 | 1.01 | 1.65 | 2.34 | 3.07 | 3.82 | 4.59 | 5.38 | 6.18 |
| .70 | 0.15 | 0.71 | 1.42 | 2.19 | 3.00 | 3.83 | 4.67 | 5.53 | 6.39 | 7.27 |
| .60 | 0.27 | 1.02 | 1.87 | 2.75 | 3.66 | 4.57 | 5.49 | 6.42 | 7.36 | 8.30 |
| .50 | 0.45 | 1.39 | 2.37 | 3.36 | 4.35 | 5.35 | 6.35 | 7.34 | 8.34 | 9.34 |
| .40 | 0.71 | 1.83 | 2.95 | 4.04 | 5.13 | 6.21 | 7.28 | 8.35 | 9.41 | 10.47 |
| .30 | 1.07 | 2.41 | 3.66 | 4.88 | 6.06 | 7.23 | 8.38 | 9.52 | 10.66 | 11.78 |
| .20 | 1.64 | 3.22 | 4.64 | 5.99 | 7.29 | 8.56 | 9.80 | 11.03 | 12.24 | 13.44 |
| .15 | 2.07 | 3.79 | 5.32 | 6.74 | 8.12 | 9.45 | 10.75 | 12.03 | 13.29 | 14.53 |
| .10 | 2.71 | 4.61 | 6.25 | 7.78 | 9.24 | 10.64 | 12.02 | 13.36 | 14.68 | 15.99 |
| .09 | 2.87 | 4.82 | 6.49 | 8.04 | 9.52 | 10.95 | 12.34 | 13.70 | 15.03 | 16.35 |
| .08 | 3.06 | 5.05 | 6.76 | 8.34 | 9.84 | 11.28 | 12.69 | 14.07 | 15.42 | 16.75 |
| .07 | 3.28 | 5.32 | 7.06 | 8.67 | 10.19 | 11.66 | 13.09 | 14.48 | 15.85 | 17.20 |
| .06 | 3.54 | 5.63 | 7.41 | 9.04 | 10.60 | 12.09 | 13.54 | 14.96 | 16.35 | 17.71 |
| .05 | 3.84 | 5.99 | 7.81 | 9.49 | 11.07 | 12.59 | 14.07 | 15.51 | 16.92 | 18.31 |
| .04 | 4.22 | 6.44 | 8.31 | 10.03 | 11.64 | 13.20 | 14.70 | 16.17 | 17.61 | 19.02 |
| .03 | 4.71 | 7.01 | 8.95 | 10.71 | 12.37 | 13.97 | 15.51 | 17.01 | 18.48 | 19.92 |
| .02 | 5.41 | 7.82 | 9.84 | 11.67 | 13.39 | 15.03 | 16.62 | 18.17 | 19.68 | 21.16 |
| .01 | 6.63 | 9.21 | 11.34 | 13.28 | 15.09 | 16.81 | 18.48 | 20.09 | 21.67 | 23.21 |
| .001 | 10.83 | 13.82 | 16.27 | 18.47 | 20.52 | 22.46 | 24.32 | 26.12 | 27.88 | 29.59 |